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TENSOR ANALYSIS OF DISLOCATION-STRESS RELATIONSHIP BASED ON THE EXTENDED DEFORMATION GRADIENT

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Abstract

The dislocation density used to estimate the magnitude of paleostress in rocks has been expressed in terms of a scalar quantity. Dislocations are classified into two types: edge dislocations and screw dislocations. However, the scalar expression of dislocations does not contain information on the type of dislocations. Therefore, we cannot see the effect of stress on the type of dislocations. In other words, we can extract the information related to the magnitude but not the orientation from previous dislocation-stress relationship. Then, we attempted to derive the tensor equation for dislocation-stress field. For this analysis, we introduced the extended deformation gradient tensor, that is, a differential geometrical expression of the ordinary deformation gradient tensor. We assumed that: (1) the higher order terms and spatial derivatives of dislocation density can be ignored; (2) the material is isotropic. We found that our tensor equation for dislocation-stress field is the square root expression of the equation derived from the experimental data of aluminum under static tension. Moreover, we found that the type of dislocation affects the stress field through the difference in the value of coefficients of the dislocation-stress relationship.

Key words: deformation, differential geometry, dislocation, paleostress, stress theory.

1. INTRODUCTION

We can use the fabrics of deformed mineral grains to determine orientations and magnitudes of the paleostress under some circumstances (Twiss and Moores, 1992). For rocks that have deformed at steady state, the magnitude of the paleostress may be determined from three different elements of the microstructural fabric: dislocation density, subgrain diameter and dynamically recrystallized grain diameter (Twiss and Moores, 1992). The dislocation-stress relationship is one of the relationships used for this determination. This is given by

$$\sigma^2 = (\mu b)^2 A\alpha , \tag{1}$$

where σ is the differential stress, μ is the shear modulus, b is the length of the Burgers vectors, A is a constant and α is a dislocation density scalar. Equation (1) has been first applied to metals (e.g., Weertman and Weertman, 1980), and to rocks (e.g., Kohlstedt *et al*., 1976; Poirier, 1985; Jung and Karato, 2001). On the other hand, the dislocation density tensor $\alpha^{\eta\chi}$, is defined as a function of the scalar α, a dislocation line vector l^η and a Burgers vector b^χ :

$$\alpha^{\eta\chi} = f(\alpha, l^\eta, b^\chi) . \tag{2}$$

The type of dislocation density is determined by geometrical relation between the dislocation line vector and the Burgers vector. When l^η is perpendicular to b^χ, i.e., $\eta \neq \chi$, $\alpha^{\eta\chi}$ is called edge dislocation density (Fig. 1). When l^η is parallel to b^χ, i.e., $\eta = \chi$, $\alpha^{\eta\chi}$ is called screw dislocation density (Fig. 1). Equation (1) does not contain information on the type of dislocation density, because it is a scalar equation. Therefore, we cannot see the effect of stress on the type of dislocations. In other words, from eq. (1) we can extract the information related to magnitude but not orientation. Then, we attempt to derive the tensional expression of eq. (1). Moreover, we consider the effect of stress on the type of dislocations based on experimental data (Shiozawa and Ohnami, 1974).

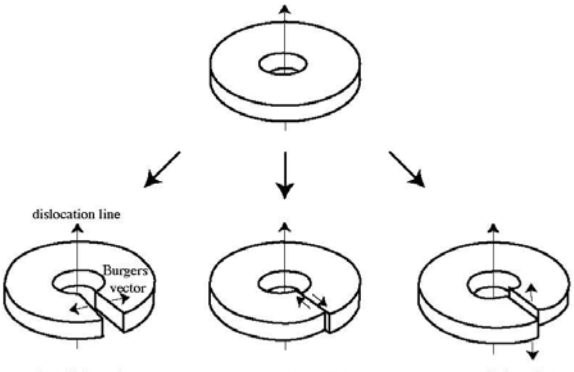

Fig. 1. Type of dislocations. The dislocation density is given by the rank-two tensor $\alpha^{\eta\chi}$. The first and second indices of $\alpha^{\eta\chi}$ are the direction of dislocation line and Burgers vector, respectively. When dislocation line is perpendicular to Burgers vector, i.e., $\eta \neq \chi$, $\alpha^{\eta\chi}$ is called the edge dislocation.When dislocation line is parallel to Burgers vector, i.e., $\eta = \chi$, $\alpha^{\eta\chi}$ is called the screw dislocation.

For this tensor analysis of a dislocation filed, we use a **d**eformation **g**radient **t**ensor (henceforth DGT). DGT is introduced by mapping from one point to the other point in a deformed medium, and is related to a displacement gradient tensor through delta function (see Section 2 for details). According to the continuum theory of defects, nonholonomic spatial change of the displacement gradient tensor is accompanied by a dislocation field (e.g., Teisseyre, 1995; Takeo and Ito, 1997; Yamasaki and Nagahama, 2002). The tensor analysis of this dislocation field is made, basing on the continuous theory of defects expressed in terms of differential geometry (e.g., Takeo and Ito, 1997). Therefore, this geometrical approach has been applied in the tensor analysis of dislocations for seismological problems such as fracture processes (e.g., Teisseyre and Czechowski, 1993; Czechowski *et al.*, 1994; Yamashita and Teisseyre, 1994), rotational ground motions (e.g., Takeo and Ito, 1997; Teisseyre *et al.*, 2004), gravity anomaly (e.g., Yamasaki and Nagahama, 1999a) and fractal fracturing (e.g., Nagahama and Teisseyre, 2000). We think that the geometrical approach to DGT is also useful for the tensor analysis of dislocation-stress field.

This paper is structured as follows. In Section 2, we concisely review the relation between **d**eformation **g**radient **t**ensor (DGT) and displacement gradient tensor. Moreover, we reconsider the differential geometrical aspects of DGT by using the previous geometrical studies such as Teisseyre (1995) and Takeo and Ito (1997). In Section 3, we extend DGT to include the effect of dislocation field by using the continuum theory of defects. In Section 4, we derive tensor equation for dislocation density and stress. Section 5 is discussion.

2. DEFORMATION GRADIENT TENSOR

The relation between the displacement gradient tensor and the DGT is concisely reviewed. We base the description of deformations on the relation between position x_i at time $t = t$ and position X_i at time $t = 0$. This is given by the following mapping:

$$x_i = x_i(X_i, t)\,. \tag{3}$$

In spatial description, displacement u_i from X_i to x_i is given by the following mapping:

$$u_i = u_i(x_i, t)\,. \tag{4}$$

Let $\det[F_{ij}]$ be the Jacobian from the coordinate system $\{X_i\}$ to $\{x_i\}$:

$$F_{ij} = \frac{\partial X_i}{\partial x_j}\,, \tag{5}$$

and $\det[E_{ij}]$ be the Jacobian from $\{u_i\}$ to $\{x_i\}$:

$$E_{ij} = \frac{\partial u_i}{\partial x_j} . \tag{6}$$

F_{ij} and E_{ij} are the deformation gradient tensor (DGT) and the displacement gradient tensor, respectively.

To connect F_{ij} with E_{ij}, we take the following translational motion:

$$X_i = x_i - u_i . \tag{7}$$

From partial derivative of eq. (7) with respect to x_j , we obtain the relation between DGT and displacement gradient tensors

$$F_{ij} = \delta_{ij} - E_{ij} \ , \tag{8}$$

where $\delta_{ij} = \partial x_i / \partial x_j$ is a delta function, i.e., $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$. To consider eq. (8) from the differential geometrical viewpoint, we introduce metric tensors defined by

$$g_{ij} = \sum_{k=1}^{3} e_{ik} e_{jk} \ , \tag{9}$$

where e_{ik} is a basis triad. From now on, we will adopt summation convention that repeated indices, such as the k above, are implicitly taken to be summed. In this convention, the above equation would simply be written as $g_{ij} = e_{ik} e_{jk}$. The metric tensor physically relates to the distortion tensor ε_{ij} (e.g., Teisseyre, 1995; Takeo and Ito, 1997):

$$\varepsilon_{ij} = \frac{1}{2}(\delta_{ij} - g_{ij}) = \frac{1}{2}(E_{ij} + E_{ji} - E_{ik} E_{jk}) \ . \tag{10}$$

Then, what is the physical meaning of basis triads e_{ik} ? Because $\delta_{ik} \delta_{jk} = \delta_{ij}$ and $\delta_{ik} E_{jk} = E_{ji}$, eq. (10) can be rewritten as

$$g_{ij} = (\delta_{ik} - E_{ik})(\delta_{jk} - E_{jk}) = F_{ik} F_{jk} \ , \tag{11}$$

where we use eq. (8) in the last step. It follows from eqs. (9) and (11) that the basis triads physically correspond to DGT:

$$e_{ij} = F_{ij} \ . \tag{12}$$

In the next section, we extend eq. (12) to include the effect of dislocations field.

3. EXTENDED DEFORMATION GRADIENT TENSOR

According to the continuum theory of defects, the material space with defect field can be treated as a non-Euclidean space (e.g., Takeo and Ito, 1997; Teisseyre, 1995; Yamasaki and Nagahama, 1999b). In the non-Euclidean space, we should distinguish contravariant tensor and covariant tensor (Fig. 2). In this case, a strict expression of eq. (9) is

$$g_{\mu\nu} = \delta_{ij} e^i_\mu e^j_\nu \ , \quad \delta^\mu_\nu = e^\mu_i e^i_\nu \ , \quad \text{and} \quad \delta^j_i = e^\mu_i e^j_\mu \ . \tag{13}$$

In the Euclidean material space where the defect does not exist, we use the coordinate system $\{x^i\}$. In the non-Euclidean material space where the defect exists, we use the coordinate system $\{q^\mu\}$. The introduction of the defect field is equivalent to transformation of the coordinates from $\{x^i\}$ to $\{q^\mu\}$ (e.g., Takeo and Ito, 1997). The concrete form of this transformation, in other words, the functional formula of the defect field depends on the related Jacobian. As previously stated in eq. (5), the Jacobian from $\{x_i\}$ to $\{X_i\}$ gives an ordinary DGT. Therefore, we deduce that the Jacobian from $\{x^i\}$ to $\{q^\mu\}$ gives DGT including the effect of the defect field. From eqs. (5) and (12), an abstract expression of this DGT is given by $e^i_\mu(q) = F'^{i}_{\mu}(q) = \partial x^i / \partial q^\mu$. In this section, we attempt to derive a concrete expression of $e^i_\mu(q) = F'^{i}_{\mu}(q)$.

Let Δx^i be the difference between $\{x^i\}$ and $\{q^\mu\}$: $q^i = x^i + \Delta x^i$. In this case, we can expand $F'^{i}_{\mu}(q) = e^i_\mu(q) = e^i_\mu(x + \Delta x)$ into the Taylor series:

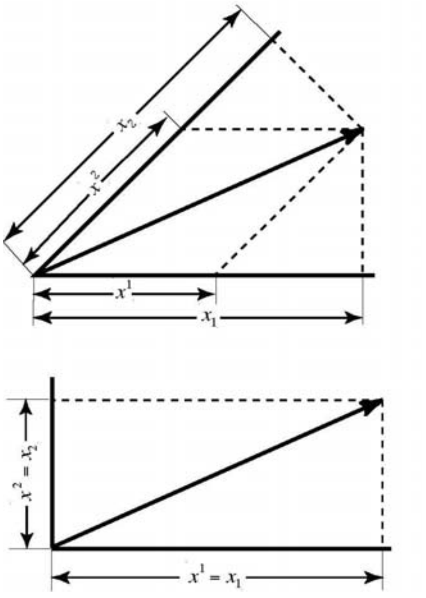

Fig. 2. Examples of contravariant and covariant component in two-dimensional space. In non-Euclidean space, the orthogonal coordinate cannot be globally put, so the oblique coordinate is needed (above figure). Contravariant components are given as the components projected in other axes in parallel, such as x^1 and x^2. Covariant components are given as the components vertically projected in the axes such as x_1 and x_2. Because neither contravariant nor covariant components are equal, it is in general necessary as shown in this figure, to distinguish these components in non-Euclidean space. In Euclidean space, the orthogonal coordinate can be put globally. Therefore, it is not necessary to distinquish the contravariant and the covariant components.

$$\begin{aligned} F'^{i}_{\mu}(q) &= e^{i}_{\mu} + \partial_{\nu} e^{i}_{\mu} \Delta x^{\nu} + \frac{1}{2} \partial_{\lambda} \partial_{\nu} e^{i}_{\mu} \Delta x^{\nu} \Delta x^{\lambda} + \cdots \\ &= e^{i}_{\kappa} (\delta^{\kappa}_{\mu} + e^{\kappa}_{j} \partial_{\nu} e^{j}_{\mu} \Delta x^{\nu} + \frac{1}{2} e^{\kappa}_{j} \partial_{\lambda} \partial_{\nu} e^{j}_{\mu} \Delta x^{\nu} \Delta x^{\lambda} + \cdots), \end{aligned} \tag{14}$$

where $e^{i}_{\mu}(x) = e^{i}_{\mu}$ and we use eq. (13). In the ordinary continuum mechanics, the gradient of DGT is ignored. In this case, all the terms related to Δx^{i} vanish, so $\{q^{\mu}\}$ becomes $\{x^{i}\}$. The defect field is interpreted here as a non-Euclidean property of the material space (e.g., Takeo and Ito, 1997; Teisseyre, 1995; Yamasaki and Nagahama, 1999b). Therefore, we should calculate the gradient of DGT to include the effect of defects field. To make this calculation short and clear, we introduce the connection defined as

$$\Gamma^{\mu}_{\lambda\kappa} = -e^{i}_{\kappa}\, \partial_{\lambda}\, e^{\mu}_{i} \ . \tag{15}$$

Connections are essential elements in the geometrical approach to physics. In the gauge theory, the connection corresponds to gauge potential such as electromagnetic potential (e.g., Peskin and Schroeder, 1995). In the theory of general relativity, gravity can be interpreted geometrically as a symmetric part of the connection $\Gamma^{\mu}_{(\lambda\kappa)} = (\Gamma^{\mu}_{\lambda\kappa} + \Gamma^{\mu}_{\kappa\lambda})/2$ (e.g., Misner *et al.*, 1973). In the continuum theory of defects, an antisymmetric part of the connection $\Gamma^{\mu}_{[\lambda\kappa]} = (\Gamma^{\mu}_{\lambda\kappa} - \Gamma^{\mu}_{\kappa\lambda})/2$ can be interpreted physically as a dislocation density tensor $\alpha^{\kappa\lambda}$ (e.g., Kröner, 1981; Takeo and Ito, 1997):

$$\alpha^{\kappa\lambda} = \varepsilon^{\kappa\mu\nu} \Gamma^{\lambda}_{[\mu\nu]} = \varepsilon^{\kappa\mu\nu} S^{\lambda}_{\mu\nu} \ , \tag{16}$$

where $S^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{[\mu\nu]}$ is called torsion tensor and $\varepsilon^{\kappa\mu\nu}$ is Levi-Civita tensor. $\varepsilon^{\kappa\mu\nu} = 0$ whenever two of the indicies coincide, and otherwise is given by $\varepsilon^{\kappa\mu\nu} = \pm 1$ if $\mathrm{sgn}\,(\kappa, \mu, \nu) = \pm 1$. For instance, $\varepsilon^{112} = 0$, $\varepsilon^{123} = 1$ and $\varepsilon^{132} = -1$. In Table 1, we summarize the correspondence between physical quantities of continuum mechanics and geometrical quantities of differential geometry. From eqs. (13) and (15), we have $e^{\kappa}_{i} \partial_{\nu} e^{i}_{\mu} = \Gamma^{\kappa}_{\nu\mu}$ and $e^{\kappa}_{j} \partial_{\lambda} \partial_{\nu} e^{j}_{\mu} = \partial_{\lambda} \Gamma^{\kappa}_{\nu\mu} + \Gamma^{\kappa}_{\lambda\sigma} \Gamma^{\sigma}_{\nu\mu}$, so, eq. (14) can be rewritten as

Table 1

One-to-one correspondence between continuum mechanics and differential geometry

Mechanics	Geometry	Equation
distorsion	metric	10
deformation gradient	basic triad	12
dislocation density	torsion	16

$$F'^{i}_{\mu}(q) = e^{i}_{\kappa}[\delta^{\kappa}_{\mu} + \Gamma^{\kappa}_{\nu\mu}\Delta x^{\nu} + \frac{1}{2}(\partial_{\lambda}\Gamma^{\kappa}_{\nu\mu} + \Gamma^{\kappa}_{\lambda\sigma}\Gamma^{\sigma}_{\nu\mu})\Delta x^{\nu}\Delta x^{\lambda} + \cdots] \ . \tag{17}$$

We divide the connection into symmetric and antisymmetric parts: $\Gamma^{\kappa}_{\nu\mu} = \Gamma^{\kappa}_{(\nu\mu)} + \Gamma^{\kappa}_{[\nu\mu]}$. The antisymmetric part is related to the dislocation density through eq. (17). We, therefore, have

$$\begin{aligned} F'^{i}_{\mu}(q) = e^{i}_{\kappa}[\delta^{\kappa}_{\mu} &+ (\Gamma^{\kappa}_{(\nu\mu)} + \frac{1}{2}\varepsilon_{\eta\nu\mu}\alpha^{\eta\kappa})\Delta x^{\nu} + \frac{1}{2}\partial_{\lambda}(\Gamma^{\kappa}_{(\nu\mu)} + \frac{1}{2}\varepsilon_{\eta\nu\mu}\alpha^{\eta\kappa})\Delta x^{\nu}\Delta x^{\lambda} \\ &+ (\Gamma^{\kappa}_{(\lambda\sigma)} + \frac{1}{2}\varepsilon_{\eta\lambda\sigma}\alpha^{\eta\kappa})(\Gamma^{\sigma}_{(\nu\mu)} + \frac{1}{2}\varepsilon_{\tau\nu\mu}\alpha^{\tau\sigma})\Delta x^{\nu}\Delta x^{\lambda} + \cdots] \ , \end{aligned} \tag{18}$$

where we use eq. (16) and $\varepsilon^{\lambda\mu\nu}\varepsilon_{\lambda\kappa\lambda} = \delta^{\mu}_{\kappa}\delta^{\nu}_{\lambda} - \delta^{\mu}_{\lambda}\delta^{\nu}_{\kappa}$. When the dislocation density vanishes, eq. (18) becomes $F'^{i}_{\mu} = e^{i}_{\kappa}\delta^{\kappa}_{\mu} = e^{i}_{\mu}$. This is eq. (12) in ordinary continuum mechanics. Then, we regard the first term as an ordinary DGT: $e^{i}_{\mu} = F^{i}_{\mu}(x)$. A derivative of dislocation density and a symmetric part of the connection give the disclination density (e.g., Yamasaki and Nagahama, 2002). In this paper, we take up only the dislocation field, so we ignore the related term. Moreover, we ignore the higher order terms of eq. (18). In this case, eq. (18) becomes

$$F'^{i}_{\mu}(q) = F^{i}_{\mu}(x) + \frac{1}{2}e^{i}{}_{\kappa}\varepsilon_{\eta\nu\mu}\alpha^{\eta\kappa}\Delta x^{\nu} \ . \tag{19}$$

We found that eq. (19) extends the DGT of eq. (12) to include the effect of dislocations. Then, in this paper, we call F'^{i}_{μ} an extended DGT.

4. DISLOCATION FIELD AND STRESS FIELD

In this section, we consider a dislocation-stress relationship. From eq. (8), we can rewrite eq. (19) in terms of displacement gradient tensor. In the Euclidean material space, the displacement tensor is related to the stress tensor, $\sigma^{\chi\gamma}$, through generalized Hooke's law: $\sigma^{\chi\gamma}(x) = C^{\chi\gamma\xi\mu}E_{\xi\mu}(x)$, where $C^{\chi\gamma\xi\mu}$ are the elastic coefficients. Therefore, we can derive the dislocation-stress relationship:

$$\sigma^{\chi\gamma}(x) = C^{\chi\gamma\xi\mu}\left[E'_{\xi\mu}(q) + \frac{1}{2}e_{\xi\kappa}\varepsilon_{\eta\nu\mu}\alpha^{\eta\kappa}\Delta x^{\nu}\right] , \tag{20}$$

where $E'^{i}_{\mu}(q) = \delta^{i}_{\mu} - F'^{i}_{\mu}(q)$. Equation (20) is general but not useful for calculations. Then, we expand eq. (20) in the particular cases as follows.

First, we assume that the material is isotropic:

$C^{\chi\gamma\xi\mu} = \lambda\delta^{\chi\gamma}\delta^{\xi\mu} + \mu(\delta^{\chi\xi}\delta^{\gamma\mu} + \delta^{\chi\mu}\delta^{\gamma\xi})$, where λ and μ are Lame's constants.

Moreover, we divide $\sigma^{\chi\gamma}$, $E'^{\chi\gamma}$ and $\alpha^{\eta\chi}$ into mean and deviatoric components:

$$\sigma^{\chi\gamma} = \tilde{\sigma}^{\chi\gamma} + \delta^{\chi\gamma}\sigma/3, \quad E'^{\chi\gamma} = \tilde{E}'^{\chi\gamma} + \delta^{\chi\gamma}E'/3 \quad \text{and} \quad \alpha^{\eta\chi} = \tilde{\alpha}^{\eta\chi} + \delta^{\eta\chi}\alpha/3 \ ,$$

where $\sigma = \sigma^{\chi}_{\chi}$, $E' = E'^{\mu}_{\mu}$ and $\alpha = \alpha^{\eta}_{\eta}$. In this case, eq. (20) gives

$$\begin{aligned} \tilde{\sigma}^{\chi\gamma}(x) &= 2\mu\tilde{E}'^{(\chi\gamma)}(q) + \mu e^{(\chi|}{}_{\kappa}\varepsilon_{\eta\nu}{}^{|\gamma)}\alpha^{\eta\kappa}\Delta x^{\nu} \\ &= 2\mu\tilde{E}'^{(\chi\gamma)}(q) + \frac{1}{3}\mu e^{(\chi|\eta}\varepsilon_{\eta\nu}{}^{|\gamma)}\alpha\Delta x^{\nu} + \mu e^{(\chi|}{}_{\kappa}\varepsilon_{\eta\nu}{}^{|\gamma)}\tilde{\alpha}^{\eta\kappa}\Delta x^{\nu} \ , \end{aligned} \tag{21}$$

$$\begin{aligned} \sigma(x) &= 3\kappa E'(q) + \frac{3}{2}\lambda e^{\mu}_{\kappa}\varepsilon_{\eta\nu\mu}\alpha^{\eta\kappa}\Delta x^{\nu} \\ &= 3\kappa E'(q) + \frac{1}{2}\lambda e^{\mu\eta}\varepsilon_{\eta\nu\mu}\alpha\Delta x^{\nu} + \frac{3}{2}\lambda e^{\mu}_{\kappa}\varepsilon_{\eta\nu\mu}\tilde{\alpha}^{\eta\kappa}\Delta x^{\nu} \ , \end{aligned} \tag{22}$$

where $E'^{(\chi\gamma)} = (E'^{\chi\gamma} + E'^{\gamma\chi})/2$, $e^{(\chi|}{}_{\kappa}\varepsilon_{\eta\nu}{}^{|\gamma)} = (e^{\chi}_{\kappa}\varepsilon^{\gamma}_{\eta\nu} + e^{\gamma}_{\kappa}\varepsilon^{\chi}_{\eta\nu})/2$ and $\kappa = \lambda + (2/3)\mu$. Axial and deviatoric dislocations corresponds to screw and edge dislocations, respectively. Therefore, the second and the third term of eqs. (21) and (22) is the contribution of screw dislocations and that of edge dislocations, respectively.

5. DISCUSSIONS

In this section, we will compare eq. (21) with equations derived by the previous studies. From eq. (21) and a Taylor series, $\tilde{\sigma}^{\chi\gamma}(x) = \tilde{\sigma}^{\chi\gamma}(q-\Delta x) = \tilde{\sigma}'^{\chi\gamma}(q) - \partial_{\nu}\tilde{\sigma}^{\chi\gamma}\Delta x^{\nu} + \cdots$, we have

$$\partial_{\nu}\tilde{\sigma}^{\chi\gamma} = -\mu e^{(\chi|}{}_{\kappa}\varepsilon_{\eta\nu}{}^{|\gamma)}\alpha^{\eta\kappa} \ . \tag{23}$$

This is a tensor expression of the following equation (Teisseyre and Czechowski, 1993):

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x} = -\mu\alpha \ . \tag{24}$$

To analyse the evolution of stresses under the earthquake premonitory and fracture rebound processes, Czechowski *et al.* (1994) used two equations: $\partial_x\sigma = f_1(\sigma, \sigma_R, V)$ and $V = f_2(\sigma, \sigma_R)$, where $f_1(\sigma, \sigma_R, V)$ and $f_2(\sigma, \sigma_R)$ are polynomial expressions of stress σ, resistance stress σ_R, and velocity of dislocations *V*. From these two equations, we have $\partial_x \sigma = f_1(\sigma, \sigma_R, f_2(\sigma, \sigma_R)) = f_3(\sigma, \sigma_R)$. Czechowski *et al.* (1994) showed $f_3(\sigma, \sigma_R) \propto \sigma^2$ in the particular case: $\sigma >> \sigma_R$. The solution of the differential equation, $\partial_x \sigma \propto \sigma^2$, is given by

$$\sigma = \frac{A'}{x} \ , \tag{25}$$

where A' is a constant. From eqs. (24) and (25), we can obtain $\sigma^2 = \mu A' \alpha$ that corresponds to eq. (1) (Czechowski *et al.*, 1994). If the work, $\int \sigma \mathrm{d}x \approx \sigma x$, is proportional to the strain energy of a dislocation μb^2, we have $A' = \sigma x \propto \mu b^2$.

The scalar equation (1) is derived by the scalar equation (24). Then, we try to derive the tensor expression of eq. (1) by using the tensor equation (23). The tensor expression of eq. (25) is

$$\tilde{\sigma}^{\chi\gamma} = \frac{A'^{\chi}}{x_{\gamma}} \ , \tag{26}$$

where A'^{χ} is an arbitrary vector. Substitution of eq. (26) into eq. (23) leads to

$$\tilde{\sigma}_{\rho\nu} \tilde{\sigma}^{\rho\gamma} = \mu A'_{\chi} e^{(x|}{}_{\kappa} \varepsilon_{\eta\nu}{}^{|\gamma)} \alpha^{\eta\kappa} \ , \tag{27}$$

where A'_{χ} is the inverse of A'^{χ}. This is a tensor expression of eq. (1). If the work $\tilde{\sigma}^{\chi\gamma} x_{\gamma}$ is proportional to the strain energy of a dislocation, $\mu b_i b^i$, we have $A'_{\chi} = \tilde{\sigma}_{\chi\gamma} x^{\gamma} = \mu b_i b^i A_{\chi}$. The right side of eq. (27) is a function of the type of dislocations, i.e., edge dislocations ($\eta \neq \chi$) and screw dislocations ($\eta = \chi$). Therefore, eq. (27) means that the stress field is related not only to the magnitude of a dislocation density, but also to the type (the orientation). To estimate the paleostress, we reverse this order. That is, theoretically, we can estimate the magnitude and the orientation of paleostresses by the observations of the dislocation density tensor. Next, we will see the effect of the dislocation type on the stress based on the experimental data.

Stress-dislocation field equation (27) is based on eq. (16) that is one of the basic equations of continuum theory of defects (e.g., Takeo and Ito, 1997; Yamasaki and Nagahama, 2002). By using the continuum theory of defects and an X-ray back reflection Laure method, Shiozawa and Ohnami (1974) made the experimental examinations of stress-dislocation field. Figure 3 shows the relation between torsion tensors and resolved shear stresses in pure commercial aluminum under static tension (the data are taken from Shiozawa and Ohnami, 1974). The equations chosen to fit the data were as follows:

$$\tau = 1.63 \times 10^{-4} \sqrt{S_{12}^{1}} + 1.21 \ , \tag{28}$$

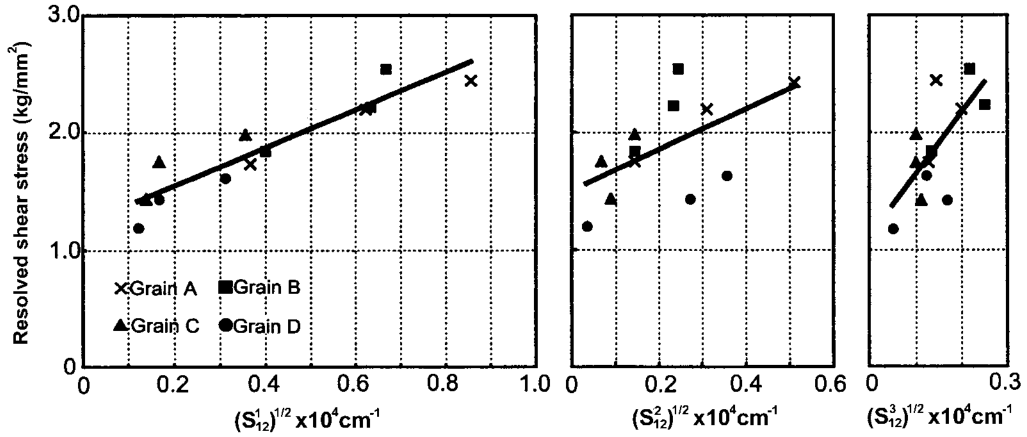

Fig. 3. The square root of torsion tensors of Grain A to D in pure commercial aluminum plotted against resolved shear stress under static tension. The data are taken from Shiozawa and Ohnami (1974). The slopes of edge dislocations S^1_{12} and S^2_{12} are 1.63 and 1.75, respectively. The slope of screw dislocation S^3_{12} is 5.31.

$$\tau = 1.75\times 10^{-4}\sqrt{S^2_{12}} + 1.50 \ , \tag{29}$$

$$\tau = 5.31\times 10^{-4}\sqrt{S^3_{12}} + 1.10 \ , \tag{30}$$

where τ is a resolved shear stress and S^i_{12} $(i = 1,2,3)$ is a torsion tensor defined as eq. (16). The second term of each equation is attributed to the frictional stress for the motion of dislocations (Shiozawa and Ohnami, 1974). From eq. (16), the torsion tensors correspond to dislocation density tensors, i.e., $S^1_{12} = \alpha^{31}$ (edge type), $S^2_{12} = \alpha^{32}$ (edge type) and $S^3_{12} = \alpha^{33}$ (screw type). Therefore, eqs. (28) to (30) show that the shear stress depends not only on the magnitude of dislocation density but also on the type (the orientation). Moreover, it is found that eqs. (28) to (30) correspond to square root expressions of eq. (27). According to Shiozawa and Ohnami (1974), $b = 2.858\times 10^{-7}$(mm) and $\mu = 2.7\times 10^{3}$(kg/mm^2). Therefore, $\sqrt{A}$-value of the edge dislocations, S^1_{12} and S^2_{12} is 0.211 and 0.227, respectively. On the other hand, $\sqrt{A}$-value of the screw dislocation, S^3_{12}, is 0.688. This means that the constant depends on the type of dislocations.

We derived eq. (27) based on the two assumptions: (1) the higher order terms and the spatial derivative of dislocation density can be ignored (Section 3); (2) the material is isotropic (Section 4). To free our model from (1), we should consider the effect of disclination density (e.g., Yamasaki and Nagahama, 2002). This means that we must treat the material space as Riemann-Cartan space (Takeo and Ito, 1997). To free our model from (2), we should calculate the elastic coefficient of each material. It is clear that we cannot apply the experimental result of aluminum to complicated materials, rocks. We need further data of the experimental relation between the dislocation density tensors (not scalars) and stress in rocks (olivine, etc).

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