Concordance of Tremor Occurrence with Poisson Distribution in the Jas-Mos Coal Mine

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Abstract
About 70 000 mining tremors recorded from 1989 to 2004 at the Jas-Mos coal mine have been analysed. The tremors had seismic energy in the range from $1 \cdot 10^3$ J to above $1 \cdot 10^6$ J.

Statistical analysis of the concordance of empirical frequency of tremor occurrence distributions with Poisson distribution has been carried out, gradually decreasing the seismic energy discrimination level. Then the distributions of interevent time intervals have been determined verifying if they approached the exponential distribution.

It is concluded that in all cases one might determine the energy discrimination levels above which the tremor occurrence approaches the Poisson process. This means that the occurrence of tremors is random, and the consequences concerning their prediction are briefly discussed.

1. Introduction

In Polish coal mines, mining seismology can be considered as a most common geophysical method. It is expected that on the basis of results of seismological observations one can evaluate rockburst hazard and would be able to predict stronger mine tremors. Therefore, the relations between induced seismicity and geological conditions as well as recent mining operations (their intensity and method) are relatively well known.

Recently one can meet concepts on the application of the Poisson process to the description of the stronger mine tremor occurrence in the areas of intense underground coal extraction. According to Gnedenko (1969) the Poisson process describing the occurrence of events in time interval from 0 to $t$ has been derived under the following assumptions:
1. The process is stationary. This means that the occurrence probability of events depends only on the numbers and time interval duration and is not changed by a time shift in all time intervals;

2. There is an absence of aftereffects. This means that the probability of occurrence of \( k \) events during time interval \((t, t+\Delta t)\) does not depend on how many times the events occurred previously or how they occurred. This signifies mutual independence of occurrence of any number of events during non-overlapping time intervals;

3. The process is orderly, sometimes referred to as orderly in a Khinchin sense. This means that the events do not occur simultaneously and it is impossible for two or several events to occur during appropriately small time interval.

Then the probability of occurrence of exactly \( n \) events in time interval of length \( t' \), \( p(n, \lambda t') \), is determined (Gnedenko 1969, Fisz 1969) as:

\[
p(n; \lambda t') = \frac{e^{-\lambda t'}(\lambda t')^n}{n!},
\]

(1)

\( n = 0, 1, 2, \ldots \) \( \lambda t' = \frac{N}{T} \) is the parameter representing the expected value of event occurrence in time interval \( t' \), and \( N \) is the total number of events in \( T \) time intervals of length \( t' \).

If the event generation process is Poissonian, then it would be purely random, memoryless and stationary (Gnedenko 1969).

The aim of the paper is an analysis of the seismic activity distribution induced by mining operation to ascertain if one can approximate it by Poisson process. It has been assumed that the conformability will be checked for the sequence of tremors from the Jas-Mos coal mine, lowering successively their discrimination seismic energy level.

2. Description of the Research Area

The Jas-Mos coal mine is located in the south-west part of the Upper Silesian Coal Basin, where the westerly oriented compression is observed in the sedimentary cover with appropriate modification induced there by movements of the basement blocks (Kotas 1985, Pierwola 1998). From the morphological point of view, there are several erosional depressions and uplifts (in the Quaternary series) with relative heights up to 30 m.

In this area, the Carboniferous deposits are strongly disturbed by tectonics as a result of the orogenic movements of the Carboniferous age with further interaction in the post-Triassic and post-Cretaceous periods.

In the tectonics of the area one may distinguish the generally dominant direction which has NNE-SSW orientation (direction of pressure). Besides this main direction, one can distinguish further two general zones dissimilar in their tectonic structure – eastern and western. The border between them is approximately along the Orłowa disturbance. The roof of the Carboniferous deposits (created by long lasting erosion
and tectonic process) is covered by impervious clay strata. The morphology of the roof is very diversified. Central and western parts of the deposit where the roof of the Carboniferous strata is 0.0 ± 190.0 m above sea level, are the most elevated. The Earth surface consisting of irregular ridges cut by relatively deep erosional valleys creates the heights with slopes inclined to the north, east and south.

Due to complicated geological structure, the Jas-Mos coal deposit belongs to the folded-faulted type zone. Taking into account the fault dimensions in the deposit, the dominant system of their strikes is meridional while the latitudinal system is created by younger dislocations.

From January 1, 1989, at the Jastrzębie coal mine (presently it is part of the Jas-Mos coal mine) the seismological observations have been started using 6-channel Górnik 1 equipment (Jakubów 2001). Since 2003 the seismological equipment has been gradually modified; it contains now:

- 6-channel Górnik 1 equipment
- 24-channel ARES seismoacoustic system
- 24-channel TSS-SO equipment
- modernised seismological recorder AS-1 (up to 32 channels)
- seismological system ARAMIS (16 channels).

3. Data Base

The data base consisted of approx. 70 000 tremors recorded from 1989 to 2004. The tremors had seismic energies above the $1 \cdot 10^3 J$ level, the largest being above $1 \cdot 10^6 J$. We have analysed more than 50% of them (~ 40 000 tremors) from the whole sequence of mine tremors, dividing them into 3 periods of seismic activity (Fig. 1):

- low seismicity (from 1 December 2000 to 1 September 2002); a total of 5617 tremors; 255.3 tremors per month, on the average,
- mean seismicity (from 1 May 1992 to 1 January 1994); a total of 9322 tremors; 443.9 tremors per month, on the average,
- high seismicity (from 1 November 1997 to 1 August 2000); a total of 26 370 tremors; 775.6 tremors per month, on the average.

For the selected seismic energy levels, the analysis of the concordance of tremor occurrence with Poisson distribution has been made in two ways:

1. by a statistical analysis of the consistence of tremor occurrence frequency with theoretical Poisson distribution,
2. by an analysis of the distribution of time duration intervals between two consecutive tremors and its consistence with exponential distribution.
4. Analysis of Results

4.1 Statistical analysis of the occurrence frequency with Poisson distribution

The observed frequencies of tremor occurrence in selected time intervals have been compared with frequencies expected from Poisson distribution determined from Eq. 1. Then the differences between empirical and theoretical distributions have been tested with $\chi^2$ test (Fish 1969, Jasiulewicz and Kordecki 2001). In all the cases, the $\chi^2$ statistics had 2 degrees of freedom ($df = 2$) calculated from:

$$df = (k - 1 - c),$$

where $k$ is the number of classes of tremor occurrence, and $c$ is the number of estimated distribution parameters. At the confidence level $\alpha = 0.05$ one can read $\chi_a^2 = 0.59915$. Therefore, the critical area is $Q = [5.9915; \infty]$. Thus, we have proceeded as follows:

1. when $\chi_{obs}^2 \geq \chi_a^2$, the hypothesis of distribution concordance was rejected,

2. when $\chi_{obs}^2 \leq \chi_a^2$, the hypothesis was accepted, that means, we had no statistical reasons to reject the hypothesis of concordance of both distributions (it is true in 90% confidence interval).

An example of statistical analysis is presented in Fig. 2.
Fig. 2. Example of statistical analysis of the empirical frequency distribution concordance with Poisson distribution for the interval of average seismicity for discrimination levels of seismic energy $E$: A) $E > 3 \cdot 10^5$ J, B) $E > 1 \cdot 10^6$ J, C) $E > 8 \cdot 10^4$ J, D) $E > 5 \cdot 10^4$ J. $N =$ number of tremors; $T =$ number of time intervals of length $t$; $\lambda t'$ = occurrence of activity in period $t'$. 
Fig. 3. Examples of the annual cumulated interevent time intervals distributions $P(\xi > t')$ for the selected years of mining tremor sequences: A – 1990 and C – 1996 (the Kolmogorov test suggests to reject the concordance with exponential distribution); B – 1991 and D – 2001 (from the results of Kolmogorov tests there are no reasons to reject the hypothesis).
Analysis of the interevent time from mine tremor sequence

The distributions of interevent time intervals have been analysed for successive years of mining (Fig. 3) as well as for the whole tremor sequence (Fig. 4). The concordance has been verified for both methods, that is:

- the distribution of interevent time intervals, and
- the comparison of the tremor occurrence frequency with the Poisson distribution.

The cumulative probability distributions $P(\xi > t')$ representing the probability that expectation time $\xi$ for tremor occurrence would be larger than $t'$ are presented in semi-logarithmic scale. The $P(\xi > t')$ values are obtained from

$$P(\xi > t') = \frac{i > t'}{i},$$

where $i > t'$ is the number of interevent time intervals with $\xi > t'$, and $i = n - 1$ is the total number of time intervals between two consecutive events.

In the figures the observed probability $P(\xi > t')$ values are denoted by lines with triangles while the straight lines (in semi-log scale) denote the exponential distribution

$$P(\xi > t') = e^{-\lambda t'},$$

for the $\lambda$ value determined from the tremor’s statistics.

From the observed distributions of time intervals in successive years one can see that in some years (e.g., 1991, 2001) the empirical distribution $P(\xi > t')$ fits the exponential distribution relatively well but in other periods (e.g., 1990, 1996) the deviation is large.

Figure 4 presents the observed distribution of $P(\xi > t')$ for the whole mining tremor sequence of 1989-2004 for the Jas-Mos coal mine. Applying the Kolmogorov test we can confirm the concordance with exponential distribution at confidence level $\alpha = 0.05$ ($\lambda = [1.4652 \pm 0.091] < 1.36$).

![Fig. 4. Cumulative probability of interevent time distribution $P(\xi > t')$ for 1989-2004 for the whole tremor sequence.](image-url)
Then we made an analysis for the selected three periods of seismicity. The results are presented in Fig. 5: left panels show the frequency of occurrence distributions and the right panels the corresponding distributions of interevent time intervals $P(\xi > t')$. The $\chi^2$ test indicates that when decreasing the seismic energy discrimination level, the value of $\chi^2$ is increasing. This means that the concordance with Poisson distribution is decreasing. The same result can be observed on the time interval distributions – the increasing deviations from the exponential distribution, which is clearly seen from the Kolmogorov test (cf. Tables 1 and 2).

From the presented figures and results it can be concluded that the empirical tremor sequence can be considered as a Poisson distribution. This means that the tremor sequence is random. The randomness of tremor sequence indicates that basing on the information contained in that sequence only, the prediction of tremor occurrence is not possible. It is also seen that decreasing seismic energy discrimination level, the concordance with Poisson distribution is getting worse. Therefore, we may expect that below some specified seismic energy level we can look for some correlations (hidden relations) between tremor occurrence.

5. Discussion

Seismicity in underground mines is under the dominant influence of mining (Gibowicz and Kijko 1994). Large mining tremors may include non-reversible destructions in underground openings as well as in some cases may have unprofitable influence on structures located on the earth surface. Therefore, their prediction appears very important. In the deterministic sense the tremor prediction should include the determination of place (focal area), time of occurrence and tremor size. To be applicable, these data should be obtained quickly (Marcak and Zuberek 1994).

The reliability of statistical estimation of the probability of tremor occurrence depends on the amount and accuracy of a priori information used in prediction scheme and on the quality of data on which the prognosis is based.

Poissonian or non-Poissonian character of mining tremors sequence is a fundamental question in the problem of short-term or long-term prediction (Lasocki 1990, Marcak and Zuberek 1994, Gibowicz and Lasocki 2001).

Lasocki (1992) testing the Poissonian distribution of mine tremor generation has found that the Poissonian character of seismicity is observed in short time sequences and should be rejected for long complete series. It was also found that the series consisting only of large events tended to be Poissonian suggesting that the prediction of large events should be based on information carried by small events (Gibowicz and Lasocki 2001). Our results from the Jas-Mos coal mine support this hypothesis and indicate that above some seismic energy level the time distribution of mining tremors is approaching the Poisson process. From the assumptions of the Poissonian character of the process, one can conclude that the tremor occurrence is purely random. Therefore, using this tremor sequence we are not able to predict their occurrence. Increasing the detection range of seismic network and decreasing the seismic energy discrimination level we can look for some relations allowing the prediction in
the way similar to Kornowski’s (2002) formulation. Further development of such models may help to find the best way to the proper formulation of the dilemma of the tremor prediction.

Fig. 5. Comparison of the frequency of occurrence distributions $p(n; \lambda t')$ and cumulated distributions of the interevent time intervals $P(\xi > t')$ for decreasing levels of seismic energy distribution $E$ (high seismicity period): A) $E > 1 \cdot 10^5$ J, B) $E > 7.5 \cdot 10^4$ J, C) $E > 5 \cdot 10^4$ J, D) $E > 2.5 \cdot 10^4$ J.
Table 1
Statistical analysis of the empirical occurrence frequency distribution of tremors with Poisson distribution using $\chi^2$ test at confidence level $\alpha = 0.05$ (in all cases $df = 2$),

$N$ – number of tremors, $T$ – number of time intervals of length $t$

<table>
<thead>
<tr>
<th>Energy $E$ [J]</th>
<th>$\chi^2$</th>
<th>$\lambda' = N/T$</th>
<th>$N$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8.6 \times 10^5$</td>
<td>1.0642</td>
<td>0.810</td>
<td>13</td>
<td>16</td>
</tr>
<tr>
<td>$7.3 \times 10^5$</td>
<td>3.7646</td>
<td>0.343</td>
<td>22</td>
<td>64</td>
</tr>
<tr>
<td>$4.5 \times 10^5$</td>
<td><strong>11.7752</strong></td>
<td>0.097</td>
<td>56</td>
<td>576</td>
</tr>
</tbody>
</table>

Low seismicity

<table>
<thead>
<tr>
<th>Energy $E$ [J]</th>
<th>$\chi^2$</th>
<th>$\lambda' = N/T$</th>
<th>$N$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.0 \times 10^5$</td>
<td>0.1092</td>
<td>0.730</td>
<td>16</td>
<td>22</td>
</tr>
<tr>
<td>$7.0 \times 10^4$</td>
<td>0.8803</td>
<td>0.510</td>
<td>34</td>
<td>66</td>
</tr>
<tr>
<td>$4.5 \times 10^4$</td>
<td>0.9522</td>
<td>0.424</td>
<td>56</td>
<td>132</td>
</tr>
<tr>
<td>$3.0 \times 10^4$</td>
<td>1.8599</td>
<td>0.377</td>
<td>83</td>
<td>220</td>
</tr>
<tr>
<td>$1.5 \times 10^4$</td>
<td><strong>9.2800</strong></td>
<td>0.257</td>
<td>172</td>
<td>669</td>
</tr>
</tbody>
</table>

Average seismicity

<table>
<thead>
<tr>
<th>Energy $E$ [J]</th>
<th>$\chi^2$</th>
<th>$\lambda' = N/T$</th>
<th>$N$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3.0 \times 10^5$</td>
<td>0.8952</td>
<td>0.558</td>
<td>24</td>
<td>43</td>
</tr>
<tr>
<td>$1.0 \times 10^5$</td>
<td>1.4004</td>
<td>0.600</td>
<td>63</td>
<td>105</td>
</tr>
<tr>
<td>$8.0 \times 10^4$</td>
<td>2.4699</td>
<td>0.470</td>
<td>100</td>
<td>210</td>
</tr>
<tr>
<td>$5.0 \times 10^4$</td>
<td><strong>9.1136</strong></td>
<td>0.251</td>
<td>161</td>
<td>641</td>
</tr>
</tbody>
</table>

High seismicity

<table>
<thead>
<tr>
<th>Energy $E$ [J]</th>
<th>$\chi^2$</th>
<th>$\lambda' = N/T$</th>
<th>$N$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.0 \times 10^5$</td>
<td>3.2397</td>
<td>0.276</td>
<td>28</td>
<td>105</td>
</tr>
<tr>
<td>$7.5 \times 10^4$</td>
<td>3.4130</td>
<td>0.250</td>
<td>44</td>
<td>175</td>
</tr>
<tr>
<td>$5.0 \times 10^4$</td>
<td>3.8397</td>
<td>0.191</td>
<td>68</td>
<td>350</td>
</tr>
<tr>
<td>$2.5 \times 10^4$</td>
<td><strong>39.2716</strong></td>
<td>0.295</td>
<td>155</td>
<td>525</td>
</tr>
</tbody>
</table>

6. Conclusions

1. On the basis of statistical analysis of mining tremor occurrence at the Jas-Mos coal mine we are able to determine discrimination levels of seismic energy $E$ for three selected periods of seismicity:
   - low seismicity $E > 3.0 \cdot 10^4$ J (6.9 tremors per month)
   - average seismicity $E > 7.0 \cdot 10^4$ J (13.4 tremors per month)
   - high seismicity $E > 5.0 \cdot 10^4$ J (5.7 tremors per month)

Above these levels, the tremor frequency occurrence approaches the Poisson process.

2. From conclusion (1) it follows that in these periods, above the specified energy discrimination levels, one cannot determine any correlations enabling unique description of tremors generation. Therefore, the tremor prediction on the basis of...
Table 2
Results of Kolmogorov tests for cumulated interevent time intervals distributions for selected three periods of seismicity at confidence level $\alpha = 0.05$ ($F_i(x)$ – empirical distribuant; $F_0(x)$ – hypothetic distribuant; $\Delta\lambda$ – measurement fault $\lambda$)

| sup $\sup | F_i(x) - F_0(x) | = D$ | $N$ | $A = D/N$ | $\Delta\lambda$ | Critical level for $\alpha = 0.05$ is $\lambda_{0.05} = 1.36$ |
|---|---|---|---|---|---|
| Whole sequence | 0.1312 | 12 | 0.4548 | 0.1003 | $H_0$ – accepted |
| | 0.2963 | 21 | 1.3579 | 0.1296 | $H_0$ – accepted |
| | 0.2986 | 55 | 2.2142 | 0.1235 | $H_0$ – rejected |
| Low seismicity | 0.1488 | 15 | 0.5763 | 0.080 | $H_0$ – accepted |
| | 0.1206 | 33 | 0.6929 | 0.043 | $H_0$ – accepted |
| | 0.0831 | 55 | 0.6159 | 0.039 | $H_0$ – accepted |
| | 0.1004 | 82 | 0.9094 | 0.043 | $H_0$ – accepted |
| | 0.1009 | 171 | 1.3193 | 0.043 | $H_0$ – rejected |
| Average seismicity | 0.0796 | 23 | 0.3818 | 0.035 | $H_0$ – accepted |
| | 0.0553 | 62 | 0.4354 | 0.036 | $H_0$ – accepted |
| | 0.0340 | 99 | 0.3427 | 0.015 | $H_0$ – accepted |
| | 0.1115 | 160 | 0.1410 | 0.041 | $H_0$ – rejected |
| High seismicity | 0.1250 | 27 | 0.6498 | 0.053 | $H_0$ – accepted |
| | 0.1646 | 43 | 1.0789 | 0.064 | $H_0$ – accepted |
| | 0.1744 | 67 | 1.4178 | 0.067 | $H_0$ – accepted |
| | 0.1982 | 154 | 2.4591 | 0.089 | $H_0$ – rejected |

Tremor sequence observations only cannot be achieved because of their random occurrence.

3. Below these seismic energy discrimination levels (this means, increasing the detectability of seismic network) one may look for some hidden rules (correlations) influencing the tremor occurrence and possibly their prediction.

4. In the tested tremor sequences the concordance of the frequency of tremor occurrence distribution with the Poisson distribution is well confirmed by the distributions of interevent time intervals. In case of their randomness, these distributions are approaching the exponential distribution.
References

Fisz, M., 1969, Rachunek prawdopodobieństwa i statystyka matematyczna, Warszawa, PWN.


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